So $\models \{A\}c\{B\}$ iff for all interpretations I, if c is executed from a state which satisfies A then if its execution terminates in a state that state will satisfy B.[1] □

Exercise 6.7 In an earlier exercise it was asked to write down an assertion $A \in \mathbf{Assn}$ with one free integer variable i expressing that i was prime. By working through the appropriate cases in the definition of the satisfaction relation $\models^I$ between states and assertions, trace out the argument that $\models^I A$ iff $I(i)$ is indeed a prime number. □

6.4 Proof rules for partial correctness

We present proof rules which generate the valid partial correctness assertions. The proof rules are syntax-directed; the rules reduce proving a partial correctness assertion of a compound command to proving partial correctness assertions of its immediate subcommands. The proof rules are often called *Hoare rules* and the proof system, consisting of the collection of rules, *Hoare logic.*

Rule for **skip***:*

$$\{A\}\mathbf{skip}\{A\}$$

Rule for assignments:

$$\{B[a/X]\}X := a\{B\}$$

Rule for sequencing:

$$\frac{\{A\}c_0\{C\} \quad \{C\}c_1\{B\}}{\{A\}c_0; c_1\{B\}}$$

Rule for conditionals:

$$\frac{\{A \wedge b\}c_0\{B\} \quad \{A \wedge \neg b\}c_1\{B\}}{\{A\}\mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1\{B\}}$$

Rule for while loops:

$$\frac{\{A \wedge b\}c\{A\}}{\{A\}\mathbf{while}\ b\ \mathbf{do}\ c\{A \wedge \neg b\}}$$

Rule of consequence:

$$\frac{\models (A \Rightarrow A') \quad \{A'\}c\{B'\} \quad \models (B' \Rightarrow B)}{\{A\}c\{B\}}$$

[1] The picture suggests, incorrectly, that the extensions of assertions A^I and B^I are disjoint; they will both always contain $\perp$, and perhaps have other states in common.